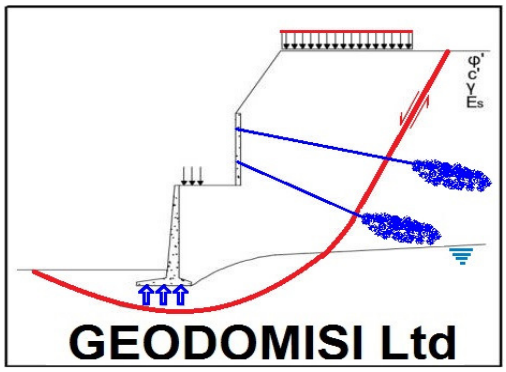
VERIFICATION OF THE ULTIMATE PUNCHING SHEAR RESISTANCE EUROCODE 2



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DESCRIPTION

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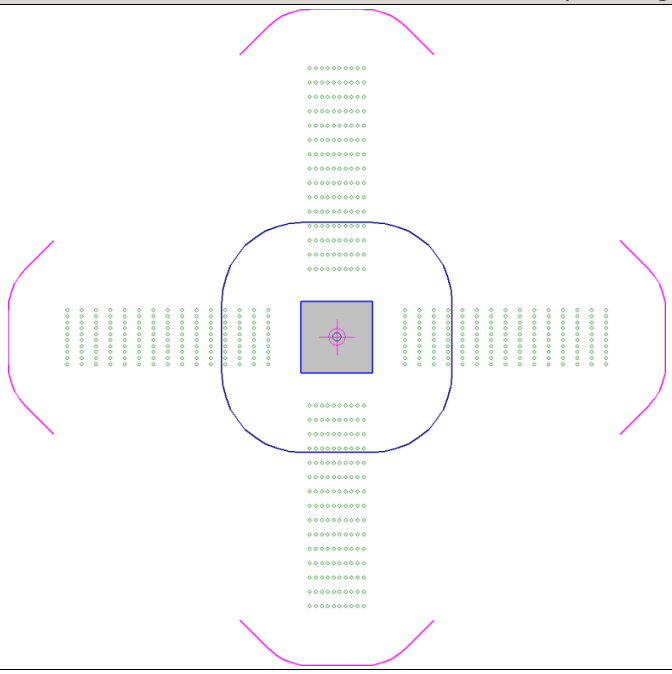

Perimeter of the support (C1)	
u_0:	1600 mm
Critical perimeter	
u_1:	4362 mm
x_G:	0 mm
y_G:	0 mm
W_{1x}:	19163.6 cm²
W_{1y}:	19163.6 cm²
Punching shear reinforcement perimeter	
$u_{out,ef}$:	5031 mm
x_G:	0 mm
y_G:	0 mm
$W_{out,ef,x}$:	51487.8 cm²
$W_{out,ef,y}$:	51487.8 cm²

CHECKS

1.- Zone adjacent to the support or load (Persistent situations)

The worst case design forces occur for load combination 1.35·SW+1.35·DL1+1.5·LL1.
The following criteria must be satisfied:

$v_{Ed} \leq v_{Rd,max}$ **2.43 MPa ≤ 4.50 MPa** ✓

Where:

v_{Ed}: Design value of the sheer stress along the control section considered. **v_{Ed}** : *2.43* MPa

$v_{Rd,max}$: Design value of the maximum punching shear resistance along the control section considered. **$v_{Rd,max}$** : *4.50* MPa

The design value of the sheer stress along the control section considered is obtained from the following expression (EN 1992-1-1:2004/AC:2008, 6.4.5):

$$v_{Ed} = \frac{|\beta \cdot V_{Ed}|}{u_0 \cdot d}$$

v_{Ed} : *2.43* MPa

Where:

V_{Ed}: Design value of the applied shear force. **V_{Ed}** : *756.75* kN

β: Coefficient which takes into account the effects of load eccentricity. (EN 1992-1-1:2004/AC:2008, 6.4.3). **β** : *1.13*

$$\beta = 1 + k_x \cdot \frac{|M_{Edx}|}{|V_{Ed}|} \cdot \frac{u_1}{W_{1x}} + k_y \cdot \frac{|M_{Edy}|}{|V_{Ed}|} \cdot \frac{u_1}{W_{1y}}$$

k_x: Coefficient which depends on the relationship between the dimensions c_y (dimension in direction of the y-axis) and c_x (dimension in direction of the x-axis) of the column (EN 1992-1-1:2004/AC:2008, Table 6.1). **k_x** : *0.60*

k_y: Coefficient which depends on the relationship between the dimensions c_x (dimension in direction of the x-axis) and c_y (dimension in direction of the y-axis) of the column (EN 1992-1-1:2004/AC:2008, Table 6.1). **k_y** : *0.60*

M_{Edx}: Design moment around the x-axis, regarding the center of gravity of the critical perimeter u_1. **M_{Edx}** : *58.65* kN·m

M_{Edy}: Design moment around the y-axis, regarding the center of gravity of the critical perimeter u_1. **M_{Edy}** : *12.30* kN·m

M_{EdOx}: Design moment around the x-axis, regarding the center of gravity of the column. **M_{EdOx}** : *58.65* kN·m

M_{EdOy}: Design moment around the y-axis, regarding the center of gravity of the column. **M_{EdOy}** : *12.30* kN·m

u_1: Critical punching shear perimeter (EN 1992-1-1:2004/AC:2008, 6.4.2). **u_1** : *4362* mm

$$W_{1x} = \int_0^{u_1} |e_y| \cdot dl$$

W_{1x} : *19163.6* cm²

dl: Differential element of the critical perimeter length.

e_y: Distance from dl to the axis where the moment M_{Edx} acts about.

$$W_{1y} = \int_0^{u_1} |e_x| \cdot dl$$

W_{1y} : *19163.6* cm²

e_x: Distance from dl to the axis where the moment M_{Edy} acts about.

u_0: Verification critical punching shear perimeter of the area adjacent to the support or load (EN 1992-1-1:2004/AC:2008, 6.4.5). **u_0** : *1600* mm

d: Nominal depth of the slab. **d** : *220* mm

The design value of the maximum punching shear resistance along the control section considered is obtained from the following expression (EN 1992-1-1:2004/AC:2008, 6.4.5):

$$v_{Rd,max} = 0.5 \cdot \nu \cdot f_{cd}$$

$v_{Rd,max}$: *4.50* MPa

$$\nu = 0.6 \cdot \left(1 - \frac{f_{ck}}{250}\right)$$

ν : *0.54*

Where:

f_{ck}: Concrete compressive strength. **f_{ck}** : *25.00* MPa

f_{cd}: Design value of the concrete compression force in the direction of the longitudinal member axis. **f_{cd}** : *16.67* MPa

2.- Zone with punching shear reinforcement (Persistent situations)

The worst case design forces occur for load combination 1.35·SW+1.35·DL1+1.5·LL1.
The following criteria must be satisfied:

$v_{Ed} \le v_{Rd,cs}$ **0.89 MPa ≤ 4.90 MPa** ✓

Where:

v_{Ed}: Design value of the sheer stress along the control section considered. **v_{Ed}** : *0.89* MPa

$v_{Rd,cs}$: Design value of the punching shear resistance of a slab with punching shear reinforcement along the control section considered. **$v_{Rd,cs}$** : *4.90* MPa

The design value of the sheer stress along the control section considered is obtained from the following expression (EN 1992-1-1:2004/AC:2008, 6.4.3):

$$v_{Ed} = \frac{|\beta \cdot V_{Ed}|}{u_1 \cdot d}$$ **v_{Ed}** : *0.89* MPa

Where:

V_{Ed}: Design value of the applied shear force. **V_{Ed}** : *756.75* kN

β: Coefficient which takes into account the effects of load eccentricity. (EN 1992-1-1:2004/AC:2008, 6.4.3). **β** : *1.13*

$$\beta = 1 + k_x \cdot \frac{|M_{Edx}|}{|V_{Ed}|} \cdot \frac{u_1}{W_{1x}} + k_y \cdot \frac{|M_{Edy}|}{|V_{Ed}|} \cdot \frac{u_1}{W_{1y}}$$

k_x: Coefficient which depends on the relationship between the dimensions c_y (dimension in direction of the y-axis) and c_x (dimension in direction of the x-axis) of the column (EN 1992-1-1:2004/AC:2008, Table 6.1). **k_x** : *0.60*

k_y: Coefficient which depends on the relationship between the dimensions c_x (dimension in direction of the x-axis) and c_y (dimension in direction of the y-axis) of the column (EN 1992-1-1:2004/AC:2008, Table 6.1). **k_y** : *0.60*

M_{Edx}: Design moment around the x-axis, regarding the center of gravity of the critical perimeter u_1. **M_{Edx}** : *58.65* kN·m

M_{Edy}: Design moment around the y-axis, regarding the center of gravity of the critical perimeter u_1. **M_{Edv}** : *12.30* kN·m

M_{EdOx}: Design moment around the x-axis, regarding the center of gravity of the column. **M_{EdOx}** : *58.65* kN·m

M_{EdOy}: Design moment around the y-axis, regarding the center of gravity of the column. **M_{EdOy}** : *12.30* kN·m

u_1: Critical punching shear perimeter (EN 1992-1-1:2004/AC:2008, 6.4.2). **u_1** : *4362* mm

$$W_{1x} = \int_0^{u_1} |e_y| \cdot dl$$ **W_{1x}** : *19163.6* cm²

dl: Differential element of the critical perimeter length.

e_y: Distance from dl to the axis where the moment M_{Edx} acts about.

$$W_{1y} = \int_0^{u_1} |e_x| \cdot dl$$

W_{1y} : *19163.6* cm²

e_x: Distance from dl to the axis where the moment M_{Edy} acts about.

d: Nominal depth of the slab. **d** : *220* mm

The design value of the punching shear resistance of a slab with punching shear reinforcement along the control section considered is obtained from the following expression (EN 1992-1-1:2004/AC:2008, 6.4.5):

$$v_{Rd,cs} = 0.75 \cdot v_{Rd,c} + 1.5 \cdot \frac{\sum \left(\frac{A_{sw}}{s_r} \cdot f_{ywd,ef} \cdot \sin\alpha \right)}{u_1}$$

$v_{Rd,cs}$: *4.90* MPa

Where:

$$v_{Rd,c} = \frac{0.18}{\gamma_c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + 0.1 \cdot \sigma_{cp}$$

$v_{Rd,c}$: *0.95* MPa

with a minimum value of:

$$v_{Rd,c,min} = 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2} + 0.1 \cdot \sigma_{cp}$$

$v_{Rd,c,min}$: *0.68* MPa

Where:

γ_c: Concrete resistance reduction coefficient. **γ_c** : *1.50*

k: Coefficient which depends on the nominal depth of 'd'. **k** : *1.95*

$$k = \left(1 + \sqrt{\frac{200}{d}}\right) \le 2$$

f_{ck}: Concrete compressive strength. **f_{ck}** : *25.00* MPa

ρ_l: Geometric steel area of the main tensile longitudinal reinforcement. **ρ_l** : *0.0132*

$$\rho_l = \sqrt{\rho_{lx} \cdot \rho_{ly}} \le 0.02$$

Where:

ρ_{lx}: Ratio in X-direction. **ρ_{lx}** : *0.0132*

ρ_{ly}: Ratio in Y-direction. **ρ_{ly}** : *0.0132*

σ_{cp}: Average axial stress on the critical verification surface (positive compression), with a maximum value of $\sigma_{cp,max}$. **σ_{cp}** : *2.00* MPa

$$\sigma_{cp,max} = 0.20 \cdot f_{cd}$$

$\sigma_{cp,max}$: *3.33* MPa

f_{cd}: Design value of the concrete compression force in the direction of the longitudinal member axis. **f_{cd}** : *16.67* MPa

A_{sw}: Total area of punching shear reinforcement within a permieter concentric with the support or loaded area.

s_r: Radial distance between two concentric perimeters of reinforcement.

α: Angle between the shear reinforcement and the plane of the slab.

Reference	A_{sw} (mm²)	s_r (mm)	α (degrees)	A_{sw}/s_r (cm²/m)
111	2262	80	45.0	282.8
111	2262	80	45.0	282.8

$f_{ywd,ef}$: Effective design strength of the punching shear reinforcement.

$$f_{ywd,ef} = 250 + 0.25 \cdot d \le f_{ywd}$$

$f_{ywd,ef}$: *305.00* MPa

f_{ywd}: Design yield strength of the shear reinforcement.

$$f_{ywd} = 0.8 \cdot f_{ywk}$$

f_{ywd} : *320.00* MPa

(EN 1992-1-1:2004/AC:2008, 6.2.3(3))

f_{ywk} : *400.00* MPa

u_1: Critical punching shear perimeter (EN 1992-1-1:2004/AC:2008, 6.4.2).

u_1 : *4362* mm

3.- External zone to the punching shear reinforecement (Persistent situations)

The worst case design forces occur for load combination 1.35·SW+1.35·DL1+1.5·LL1.
The following criteria must be satisfied:

$v_{Ed} \le v_{Rd,c}$ **0.72 MPa ≤ 0.95 MPa** ✓

Where:

v_{Ed}: Design value of the sheer stress along the control section considered. **v_{Ed}** : *0.72* MPa

$v_{Rd,c}$: Design value of the punching shear resistance of a slab without punching shear reinforcement along the control section considered. **$v_{Rd,c}$** : *0.95* MPa

The design value of the sheer stress along the control section considered is obtained from the following expression (EN 1992-1-1:2004/AC:2008, 6.4.5):

$$v_{Ed} = \frac{|\beta \cdot V_{Ed}|}{u_{out,ef} \cdot d}$$

v_{Ed} : *0.72* MPa

Where:

V_{Ed}: Design value of the applied shear force. **V_{Ed}** : *756.75* kN

β: Coefficient which takes into account the effects of load eccentricity. (EN 1992-1-1:2004/AC:2008, 6.4.3). **β** : *1.05*

$$\beta = 1 + k_x \cdot \frac{|M_{Edx}|}{|V_{Ed}|} \cdot \frac{u_{out,ef}}{W_{out,ef,x}} + k_y \cdot \frac{|M_{Edy}|}{|V_{Ed}|} \cdot \frac{u_{out,ef}}{W_{out,ef,y}}$$

k_x: Coefficient which depends on the relationship between the dimensions c_y (dimension in direction of the y-axis) and c_x (dimension in direction of the x-axis) of the column (EN 1992-1-1:2004/AC:2008, Table 6.1). **k_x** : *0.60*

k_y: Coefficient which depends on the relationship between the dimensions c_x (dimension in direction of the x-axis) and c_y (dimension in direction of the y-axis) of the column (EN 1992-1-1:2004/AC:2008, Table 6.1). **k_y** : *0.60*

M_{Edx}: Design moment around the x-axis, regarding the center of gravity of the critical perimeter $u_{out,ef}$. **M_{Edx}** : *58.65* kN·m

M_{Edy}: Design moment around the y-axis, regarding the center of gravity of the critical perimeter $u_{out,ef}$. **M_{Edy}** : *12.30* kN·m

M_{EdOx}: Design moment around the x-axis, regarding the center of gravity of the column. **M_{EdOx}** : *58.65* kN·m

M_{EdOy}: Design moment around the y-axis, regarding the center of gravity of the column. **M_{EdOy}** : *12.30* kN·m

$u_{out,ef}$: Critical punching shear perimeter outside the reinforced zone (EN 1992-1-1:2004/AC:2008, 6.4.5). **$u_{out,ef}$** : *5031* mm

$$W_{out,ef,x} = \int_0^{u_{out,ef}} |e_y| \cdot dl$$

$W_{out,ef,x}$: *51487.8* cm²

dl: Differential element of the critical perimeter length.

e_y: Distance from dl to the axis where the moment M_{Edx} acts about.

$$W_{out,ef,y} = \int_0^{u_{out,ef}} |e_x| \cdot dl$$

$W_{out,ef,y}$: *51487.8* cm²

e_x: Distance from dl to the axis where the moment M_{Edy} acts about.

d: Nominal depth of the slab. **d** : *220* mm

The design value of the punching shear resistance of a slab without punching shear reinformcement along the control section considered is obtained from the following expression (EN 1992-1-1:2004/AC:2008, 6.4.4):

$$v_{Rd,c} = \frac{0.18}{\gamma_c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + 0.1 \cdot \sigma_{cp}$$

$v_{Rd,c}$: *0.95* MPa

with a minimum value of:

$$v_{Rd,c,min} = 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2} + 0.1 \cdot \sigma_{cp}$$

$v_{Rd,c,min}$: *0.68* MPa

Where:

γ_c: Concrete resistance reduction coefficient. **γ_c** : *1.50*

k: Coefficient which depends on the nominal depth of 'd'. **k** : *1.95*

$$k = \left(1 + \sqrt{\frac{200}{d}}\right) \le 2$$

f_{ck}: Concrete compressive strength. **f_{ck}** : *25.00* MPa

ρ_l: Geometric steel area of the main tensile longitudinal reinforcement. **ρ_l** : *0.0132*

$$\rho_l = \sqrt{\rho_{lx} \cdot \rho_{ly}} \le 0.02$$

Where:

ρ_{lx}: Ratio in X-direction. **ρ_{lx}** : *0.0132*

ρ_{ly}: Ratio in Y-direction. **ρ_{ly}** : *0.0132*

σ_{cp}: Average axial stress on the critical verification surface (positive compression), with a maximum value of $\sigma_{cp,max}$. **σ_{cp}** : *2.00* MPa

$$\sigma_{cp,max} = 0.20 \cdot f_{cd}$$ **$\sigma_{cp,max}$** : *3.33* MPa

f_{cd}: Design value of the concrete compression force in the direction of the longitudinal member axis. **f_{cd}** : *16.67* MPa

4.- Punching shear reinforcement (EN 1992-1-1:2004/AC:2008, 9.4.3(2))

Where shear reinforcement is required the area of a link leg (or equivalent), $A_{sw,min}$, is given by Expression (9.11).

$$A_{sw,min} \cdot (1,5 \cdot \sin\alpha + \cos\alpha) / (s_r \cdot s_t) \ge 0,08 \cdot \sqrt{(f_{ck})} / f_{yk} \qquad (9.11)$$ ✓

$$\rho_w = A_{sw,min} \cdot (1,5 \cdot \sin\alpha + \cos\alpha) / (s_r \cdot s_t)$$

$$\rho_{w,min} = 0,08 \cdot \sqrt{(f_{ck})} / f_{yk}$$

Reference	A_{sw} (mm²)	s_r (mm)	s_t (mm)	α (degrees)	ρ_w	$\rho_{w,min}$	$\rho_w \ge \rho_{w,min}$
111	113	80	33	45.0	0.0757	0.0010	✓
111	113	80	33	45.0	0.0757	0.0010	✓

where:

A_{sw}: the area of a link leg (or equivalent).

α: is the angle between the shear reinforcement and the main steel (i.e. for vertical links α = 90° and sin α = 1).

s_r: is the spacing of shear links in the radial direction.

s_t: is the spacing of shear links in the tangential direction.

f_{ck}: is in MPa **f_{ck}** : *25.00* MPa

5.- Clear distance between two isolated consecutive bars

The horizontal and vertical clear spacing d_l between two consecutive bars should be greater than or equal to s_{min} (EN 1992-1-1:2004/AC:2008, 8.2(2)):

$d_l \ge s_{min}$ **21 mm ≥ 20 mm** ✓

Where:

s_{min}: Maximum value of s_1, s_2, s_3. **s_{min}** : *20* mm

$s_1 = \varnothing_{max}$ **s_1** : *12* mm

$s_2 = 5 + d_g$ **s_2** : *17* mm

$s_3 = 20mm$ **s_3** : *20* mm

Where:

d_g: Maximum size of aggregate. **d_g** : *12* mm

$Ø_{max}$: Diameter of the thickest bar of the transverse reinforcement. **$Ø_{max}$** : *12* mm

	d_l (mm)	s_{min} (mm)	$Ø_{max}$ (mm)	
111	**21**	**20**	**12**	✓
111	21	20	12	✓

6.- Distance between the support's face and the first punching shear reinforcement

The distance between the face of the support or loaded area and the first punching shear reinforcement should not be greater than s_{max} (EN 1992-1-1:2004/AC:2008, 9.4.3):

$d_l \le s_{max}$ **80 mm ≤ 110 mm** ✓

Where:

$s_{max} = 0.5 \cdot d$ **s_{max}** : *110* mm

d: Nominal depth of the slab. **d** : *220* mm

7.- Distance between transverse consecutive reinforcement perimeters

The distance d_l between consecutive transverse reinforcement perimeters should be, at most, equal to s_{max} (EN 1992-1-1:2004/AC:2008, 9.4.3):

$d_l \le s_{max}$ **80 mm ≤ 165 mm** ✓

Where:

$s_{max} = 0.75 \cdot d$

s_{max} : *165* mm

d: Nominal depth of the slab.

d : *220* mm

8.- Distance between two consecutive reinforcements in peripheral direction

The distance d_l between two consecutive perimeter reinforcements should not be greater than s_{max} (EN 1992-1-1:2004/AC:2008, 9.4.3):

$d_l \leq s_{max}$

33 mm ≤ 330 mm ✓

Where:

$s_{max} = 1.5 \cdot d$

s_{max} : *330* mm

d: Nominal depth of the slab.

d : *220* mm

9.- Distance between the support's external face and the outermost bar inclined at 45°

This combination does not proceed since the reinforcement is located between the external faces of the support.